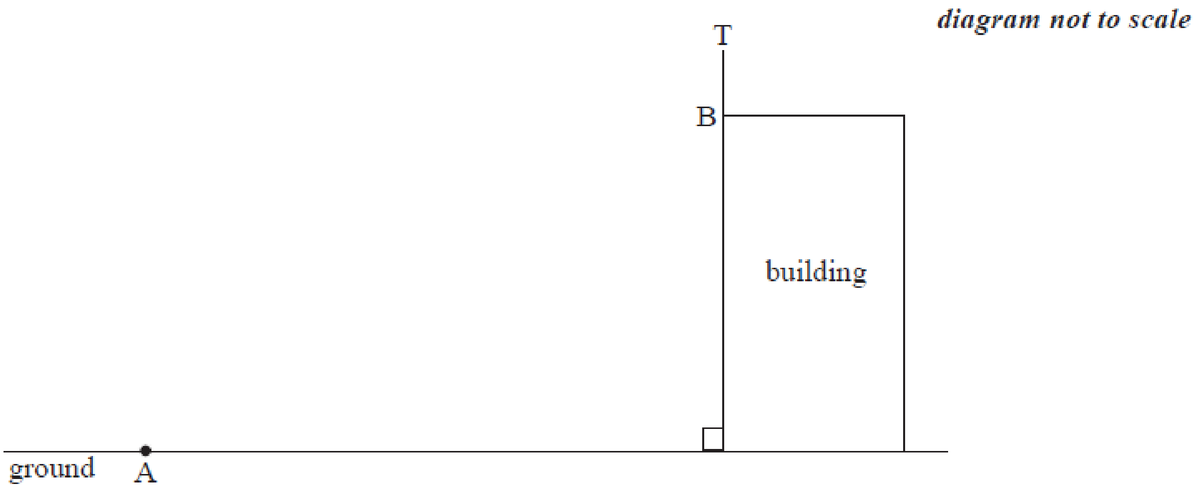
SL Paper 2

The following diagram shows a pole BT 1.6 m tall on the roof of a vertical building.

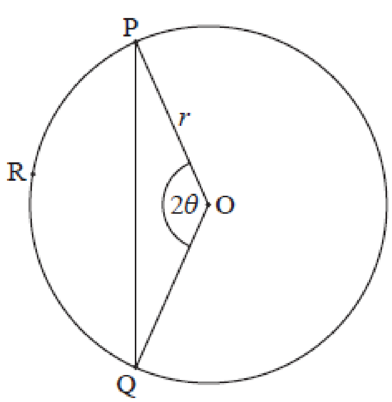
The angle of depression from T to a point A on the horizontal ground is 35° .

The angle of elevation of the top of the building from A is 30° .



Find the height of the building.

Consider the following circle with centre O and radius r .



The points P, R and Q are on the circumference, $\mathrm{P\hat{O}Q} = 2\theta$, for $0 < \theta < \frac{\pi}{2}$.

a. Use the cosine rule to show that $\mathrm{PQ} = 2r\sin\theta$. [4]

b. Let l be the length of the arc PRQ . [5]

Given that $1.3\text{PQ} - l = 0$, find the value of θ .

c(i) and (ii). Consider the function $f(\theta) = 2.6\sin\theta - 2\theta$, for $0 < \theta < \frac{\pi}{2}$. [4]

(i) Sketch the graph of f .

(ii) Write down the root of $f(\theta) = 0$.

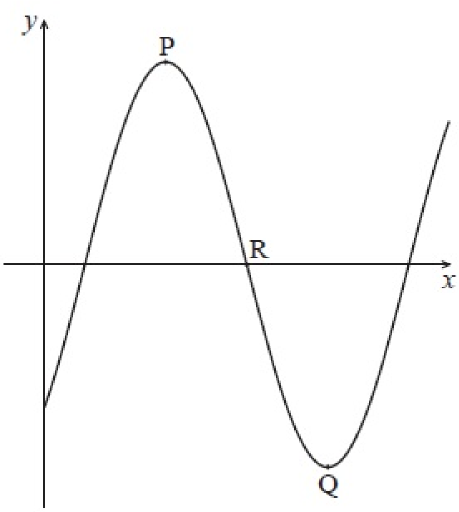
d. Use the graph of f to find the values of θ for which $l < 1.3\text{PQ}$. [3]

Consider the triangle ABC, where AB = 10 , BC = 7 and $\text{C}\widehat{\text{A}}\text{B} = 30^\circ$.

a. Find the two possible values of $\text{A}\widehat{\text{C}}\text{B}$. [4]

b. Hence, find $\text{A}\widehat{\text{B}}\text{C}$, given that it is acute. [2]

Let $f(x) = a\cos(b(x - c))$. The diagram below shows part of the graph of f , for $0 \le x \le 10$.



The graph has a local maximum at P(3, 5) , a local minimum at Q(7, − 5) , and crosses the x-axis at R.

a(i) and (ii). Write down the value of [2]

(i) a ;

(ii) c .

b. Find the value of b . [2]

c. Find the x-coordinate of R. [2]

The height, h metros, of a seat on a Ferris wheel after t minutes is given by

$$h(t) = -15\cos 1.2t + 17,\ \text{for } t \geqslant 0.$$

a. Find the height of the seat when $t = 0$. [2]

b. The seat first reaches a height of 20 m after k minutes. Find k. [3]

c. Calculate the time needed for the seat to complete a full rotation, giving your answer correct to one decimal place. [3]

The diagram below shows a circle centre O, with radius r. The length of arc ABC is 3π cm and $\text{A}\widehat{\text{O}}\text{C} = \frac{2\pi}{9}$.

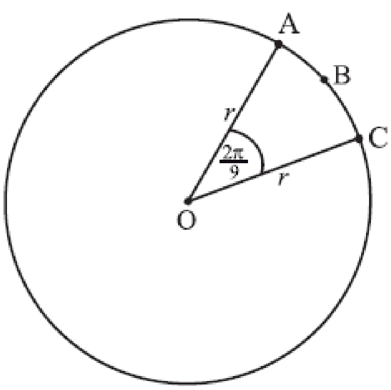


diagram not to scale

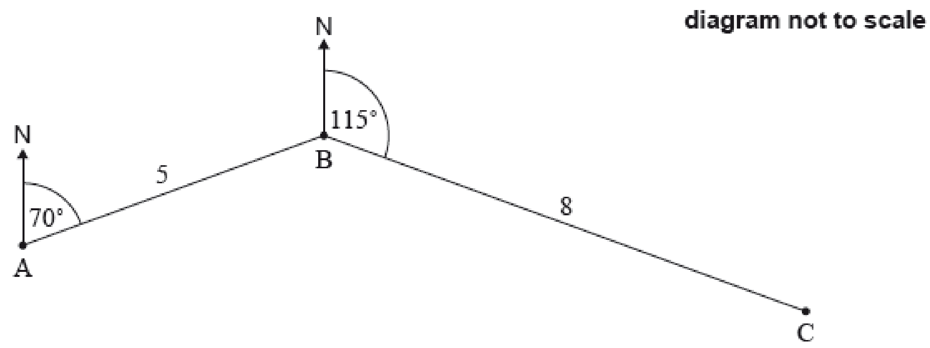
a. Find the value of r. [2]

b. Find the perimeter of sector OABC. [2]

c. Find the area of sector OABC. [2]

Triangle ABC has a = 8.1 cm, b = 12.3 cm and area 15 cm^2. Find the largest possible perimeter of triangle ABC.

The following diagram shows three towns A, B and C. Town B is 5 km from Town A, on a bearing of 070°. Town C is 8 km from Town B, on a bearing of 115°.

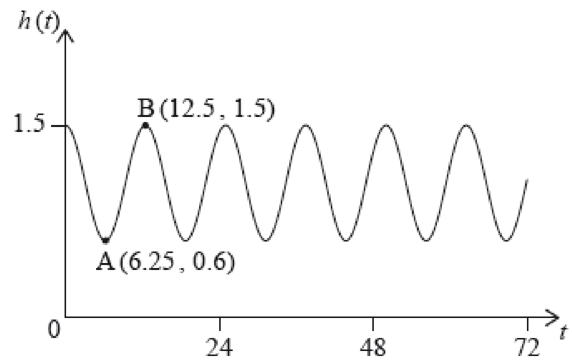


a. Find $\hat{ABC}$. [2]

b. Find the distance from Town A to Town C. [3]

c. Use the sine rule to find $\hat{ACB}$. [2]

At Grande Anse Beach the height of the water in metres is modelled by the function $h(t) = p\cos(q \times t) + r$, where t is the number of hours after 21:00 hours on 10 December 2017. The following diagram shows the graph of h , for $0 \leqslant t \leqslant 72$.



The point $A(6.25,\ 0.6)$ represents the first low tide and $B(12.5,\ 1.5)$ represents the next high tide.

a.i. How much time is there between the first low tide and the next high tide? [2]

a.ii. Find the difference in height between low tide and high tide. [2]

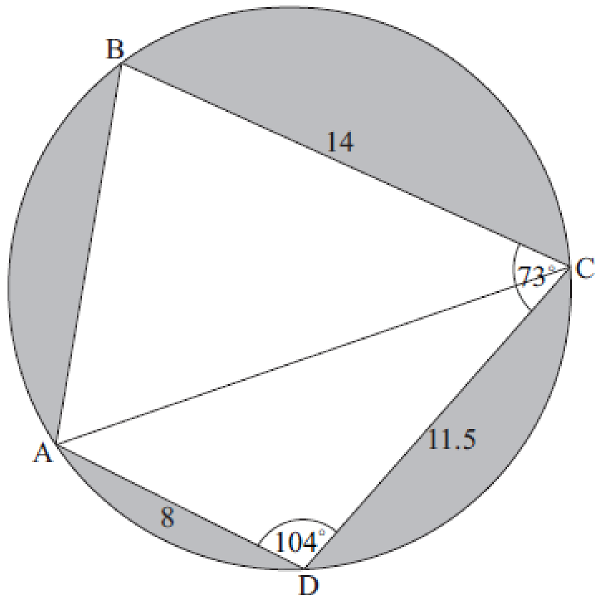
b.i. Find the value of p; [2]

b.ii. Find the value of q; [3]

b.iii. Find the value of r. [2]

c. There are two high tides on 12 December 2017. At what time does the second high tide occur? [3]

The diagram shows a circle of radius 8 metres. The points ABCD lie on the circumference of the circle.



BC = 14 m, CD = 11.5 m, AD = 8 m, $A\hat{D}C = 104^\circ$, and $B\hat{C}D = 73^\circ$.

a. Find AC. [3]

b. (i) Find $A\hat{C}D$. [5]

(ii) Hence, find $A\hat{C}B$.

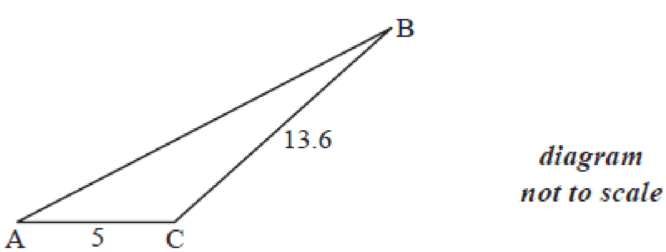
c. Find the area of triangle ADC. [2]

cd.(c) Find the area of triangle ADC. [6]

(d) Hence or otherwise, find the total area of the shaded regions.

d. Hence or otherwise, find the total area of the shaded regions. [4]

The following diagram shows the triangle ABC.

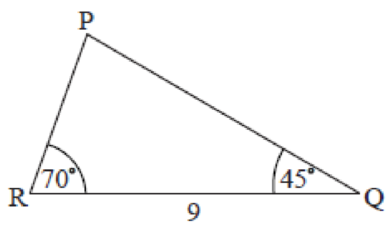


The angle at C is obtuse, AC $= 5$ cm, BC $= 13.6$ cm and the area is 20 cm^2 .

a. Find $\text{A}\widehat{\text{C}}\text{B}$. [4]

b. Find AB. [3]

The following diagram shows ΔPQR , where RQ = 9 cm, $P\hat{R}Q = 70^\circ$ and $P\hat{Q}R = 45^\circ$.

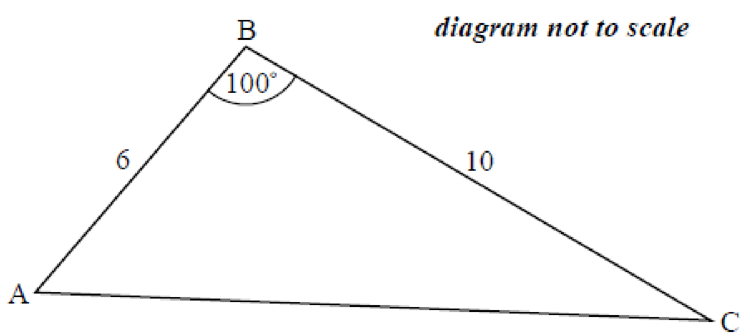


a. Find $R\hat{P}Q$. [1]

b. Find PR . [3]

c. Find the area of ΔPQR . [2]

The following diagram shows triangle ABC.



$AB = 6\,\text{cm}$, $BC = 10\,\text{cm}$, and $A\hat{B}C = 100^\circ$.

a. Find AC. [3]

b. Find $B\hat{C}A$. [3]

Two points P and Q have coordinates (3, 2, 5) and (7, 4, 9) respectively.

Let $\vec{PR} = 6\boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{k}$.

a.i. Find $\vec{PQ}$. [2]

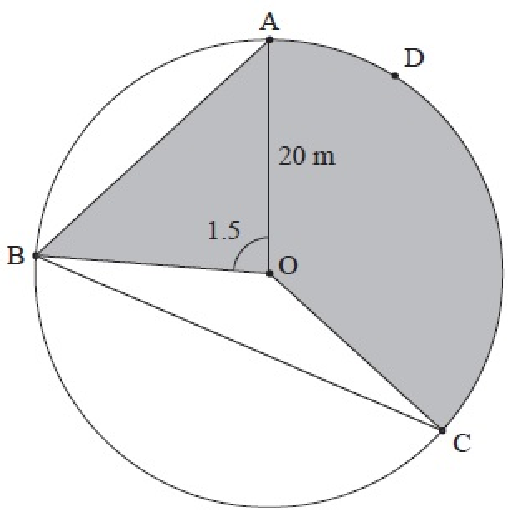
a.ii.Find $\left|\vec{PQ}\right|$. [2]

b. Find the angle between PQ and PR. [4]

c. Find the area of triangle PQR. [2]

d. Hence or otherwise find the shortest distance from R to the line through P and Q. [3]

The following diagram shows a circular play area for children.



The circle has centre O and a radius of 20 m, and the points A, B, C and D lie on the circle. Angle AOB is 1.5 radians.

a. Find the length of the chord [AB]. [3]

b. Find the area of triangle AOB. [2]

c. Angle BOC is 2.4 radians. [3]

Find the length of arc ADC.

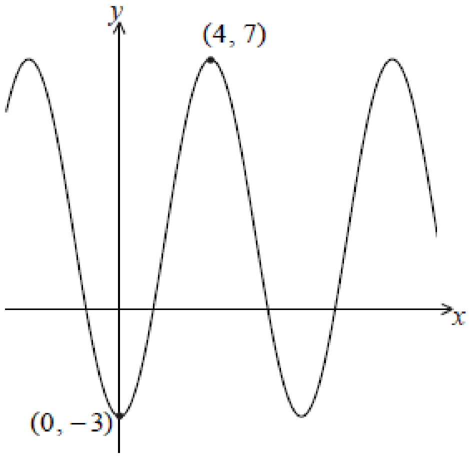
d. Angle BOC is 2.4 radians. [3]

Find the area of the shaded region.

e. Angle BOC is 2.4 radians. [4]

The shaded region is to be painted red. Red paint is sold in cans which cost \$32 each. One can covers 140 m^2. How much does it cost to buy the paint?

The graph of $y = p\cos qx + r$, for $-5 \leq x \leq 14$, is shown below.



There is a minimum point at (0, −3) and a maximum point at (4, 7) .

a(i), (ii) and (iii). Find the value of [6]

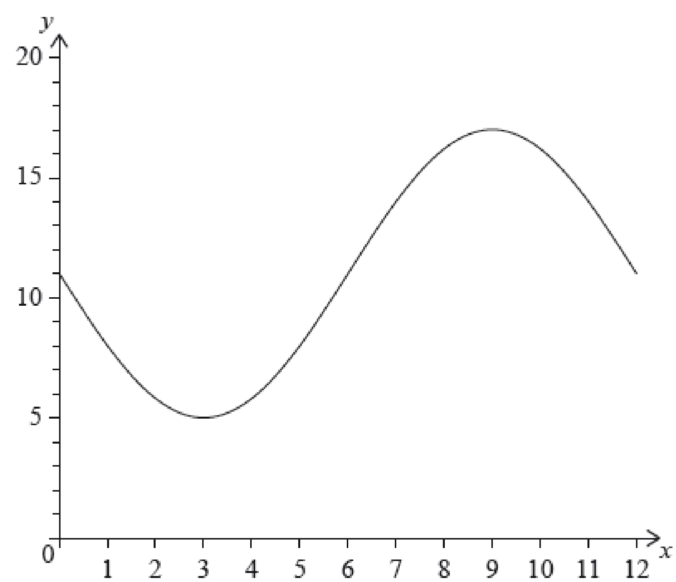
(i) p ;

(ii) q ;

(iii) r.

b. The equation $y = k$ has exactly **two** solutions. Write down the value of k. [1]

The following diagram shows the graph of $f(x) = a \sin bx + c$, for $0 \leqslant x \leqslant 12$.



The graph of f has a minimum point at $(3,\ 5)$ and a maximum point at $(9,\ 17)$.

The graph of g is obtained from the graph of f by a translation of $\begin{pmatrix} k \\ 0 \end{pmatrix}$. The maximum point on the graph of g has coordinates $(11.5,\ 17)$.

The graph of g changes from concave-up to concave-down when $x = w$.

a. (i) Find the value of c. [6]

(ii) Show that $b = \frac{\pi}{6}$.

(iii) Find the value of a.

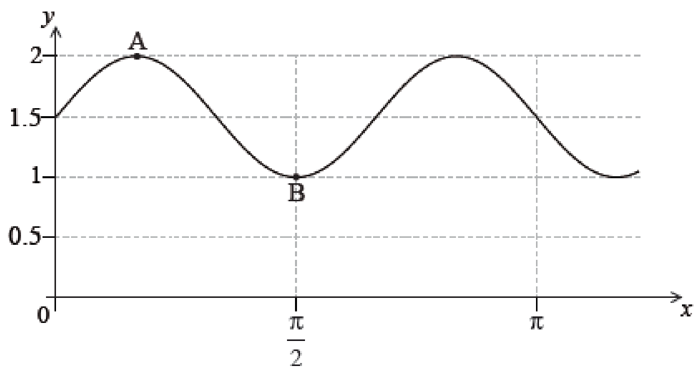
b. (i) Write down the value of k. [3]

(ii) Find $g(x)$.

c. (i) Find w. [6]

(ii) Hence or otherwise, find the maximum positive rate of change of g.

The following diagram shows part of the graph of $y = p\sin(qx) + r$.



The point $\mathrm{A}\left(\frac{\pi}{6},\ 2\right)$ is a maximum point and the point $\mathrm{B}\left(\frac{\pi}{6},\ 1\right)$ is a minimum point.

Find the value of

a. p; [2]

b. r; [2]

c. q. [2]

Note: In this question, distance is in millimetres.

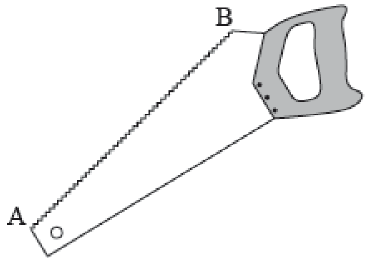
Let $f(x) = x + a\sin\left(x - \frac{\pi}{2}\right) + a$, for $x \geqslant 0$.

The graph of f passes through the origin. Let P_k be any point on the graph of f with x-coordinate $2k\pi$, where $k \in \mathbb{N}$. A straight line L passes through all the points P_k.

Diagram 1 shows a saw. The length of the toothed edge is the distance AB.

Diagram 1

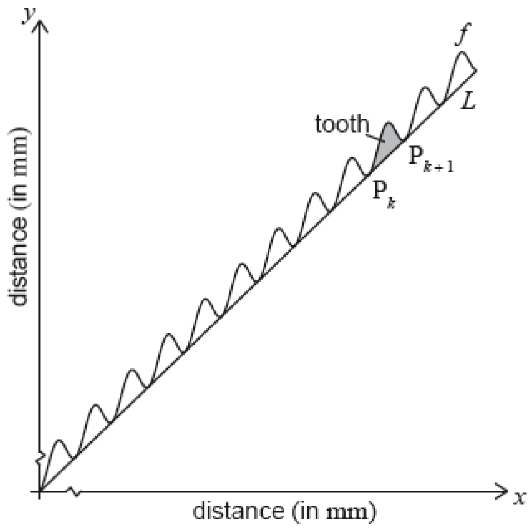
diagram not to scale



The toothed edge of the saw can be modelled using the graph of f and the line L. Diagram 2 represents this model.

Diagram 2

diagram not to scale



The shaded part on the graph is called a tooth. A tooth is represented by the region enclosed by the graph of f and the line L, between P_k and P_{k+1}.

a. Show that $f(2\pi) = 2\pi$. [3]

b.i. Find the coordinates of P_0 and of P_1. [3]

b.ii. Find the equation of L. [3]

c. Show that the distance between the x-coordinates of P_k and P_{k+1} is 2π. [2]

d. A saw has a toothed edge which is 300 mm long. Find the number of complete teeth on this saw. [6]

Let $f(x) = 3\sin x + 4\cos x$, for $-2\pi \le x \le 2\pi$.

a. Sketch the graph of f. [3]

b. Write down [3]

(i) the amplitude;

(ii) the period;

(iii) the x-intercept that lies between $-\frac{\pi}{2}$ and 0.

c. Hence write $f(x)$ in the form $p\sin(qx + r)$. [3]

d. Write down one value of x such that $f'(x) = 0$. [2]

e. Write down the two values of k for which the equation $f(x) = k$ has exactly two solutions. [2]

f. Let $g(x) = \ln(x+1)$, for $0 \le x \le \pi$. There is a value of x, between 0 and 1, for which the gradient of f is equal to the gradient of g. Find this value of x. [5]

Let $f(x) = 5\cos\frac{\pi}{4}x$ and $g(x) = -0.5x^2 + 5x - 8$ for $0 \le x \le 9$.

a. On the same diagram, sketch the graphs of f and g . [3]

b. Consider the graph of f . Write down [4]

(i) the x-intercept that lies between $x = 0$ and $x = 3$;

(ii) the period;

(iii) the amplitude.

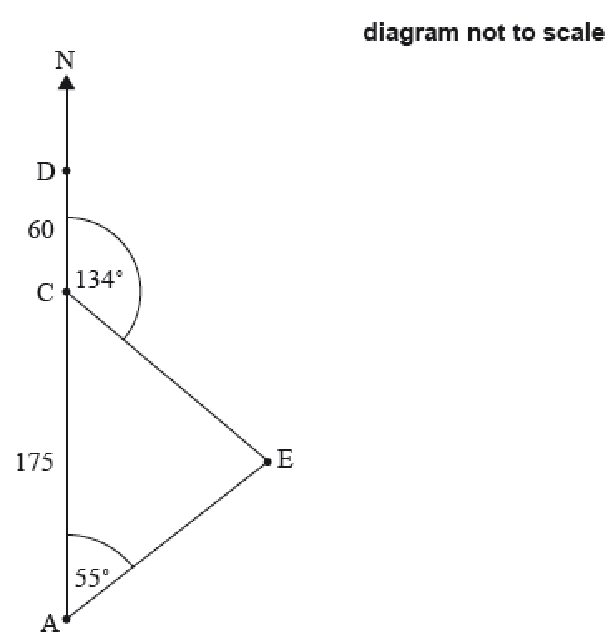
c. Consider the graph of g . Write down [3]

(i) the two x-intercepts;

(ii) the equation of the axis of symmetry.

d. Let R be the region enclosed by the graphs of f and g . Find the area of R. [5]

A ship is sailing north from a point A towards point D. Point C is 175 km north of A. Point D is 60 km north of C. There is an island at E. The bearing of E from A is 055°. The bearing of E from C is 134°. This is shown in the following diagram.



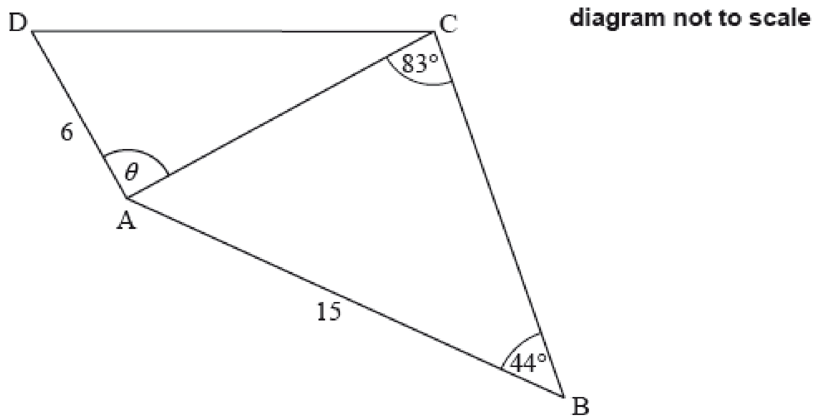
a. Find the bearing of A from E. [2]

b. Finds CE. [5]

c. Find DE. [3]

d. When the ship reaches D, it changes direction and travels directly to the island at 50 km per hour. At the same time as the ship changes direction, a boat starts travelling to the island from a point B. This point B lies on (AC), between A and C, and is the closest point to the island. The ship and the boat arrive at the island at the same time. Find the speed of the boat. [5]

The following diagram shows the quadrilateral $ABCD$.



$$\mathrm{AD} = 6 \text{ cm},\ \mathrm{AB} = 15 \text{ cm}, \mathrm{A\hat{B}C} = 44°, \mathrm{A\hat{C}B} = 83° \text{and} \mathrm{D\hat{A}C} = \theta$$

a. Find AC. [3]

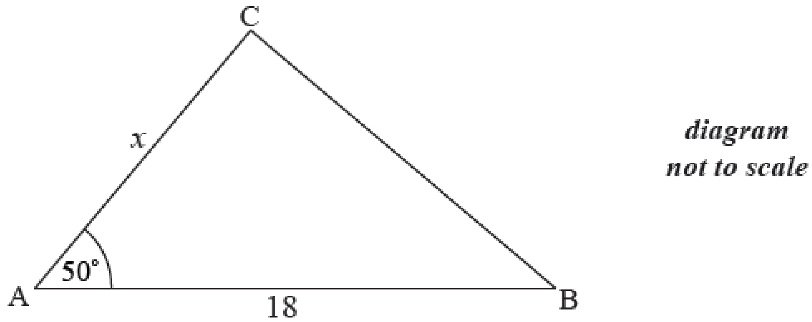
b. Find the area of triangle ABC. [3]

c. The area of triangle ACD is half the area of triangle ABC. [5]

Find the possible values of θ.

d. Given that θ is obtuse, find CD. [3]

The following diagram shows a triangle ABC.



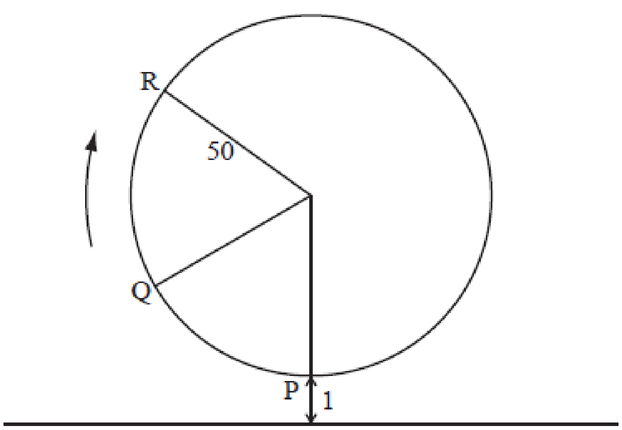
The area of triangle ABC is 80 cm^2 , AB $= 18$ cm , AC $= x$ cm and $\text{B}\hat{\text{A}}\text{C} = 50°$.

a. Find x . [3]

b. Find BC. [3]

The following diagram represents a large Ferris wheel at an amusement park.

The points P, Q and R represent different positions of a seat on the wheel.



The wheel has a radius of 50 metres and rotates clockwise at a rate of one revolution every 30 minutes.

A seat starts at the lowest point P, when its height is one metre above the ground.

a. Find the height of a seat above the ground after 15 minutes. [2]

b. After six minutes, the seat is at point Q. Find its height above the ground at Q. [5]

c. The height of the seat above ground after t minutes can be modelled by the function $h(t) = 50\sin(b(t - c)) + 51$. [6]

Find the value of b and of c .

d. The height of the seat above ground after t minutes can be modelled by the function $h(t) = 50\sin(b(t - c)) + 51$. [3]

Hence find the value of t the first time the seat is 96 m above the ground.

The depth of water in a port is modelled by the function $d(t) = p\cos qt + 7.5$, for $0 \leqslant t \leqslant 12$, where t is the number of hours after high tide.

At high tide, the depth is 9.7 metres.

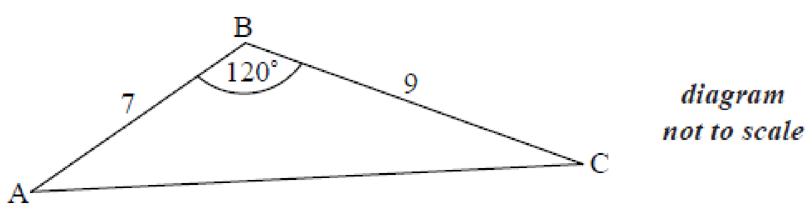
At low tide, which is 7 hours later, the depth is 5.3 metres.

a. Find the value of p. [2]

b. Find the value of q. [2]

c. Use the model to find the depth of the water 10 hours after high tide. [2]

The following diagram shows triangle ABC .

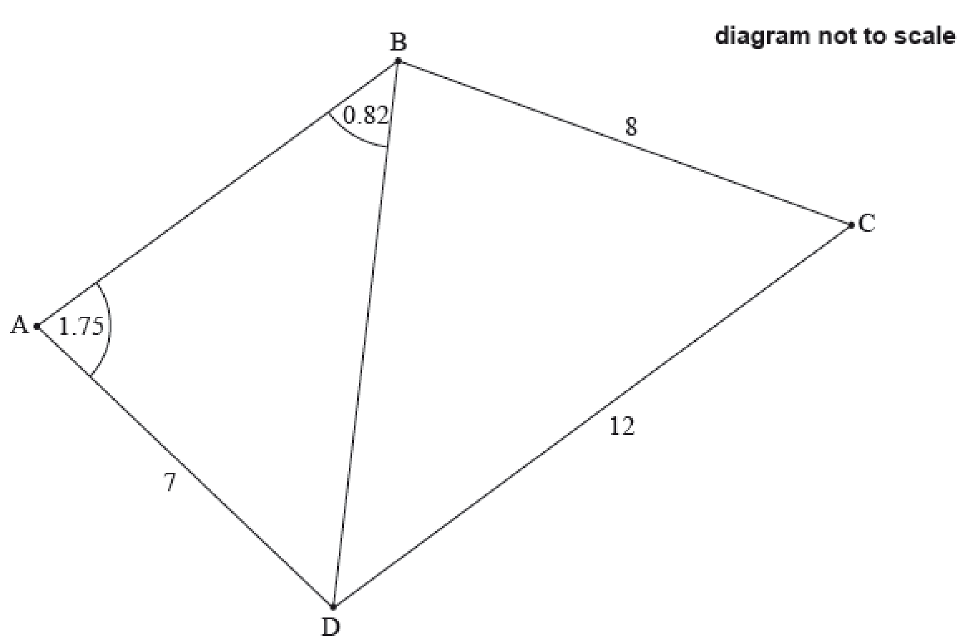


AB = 7 cm, BC = 9 cm and $\mathrm{A\hat{B}C} = 120^\circ$.

a. Find AC . [3]

b. Find $\mathrm{B\hat{A}C}$. [3]

The following diagram shows a quadrilateral ABCD.

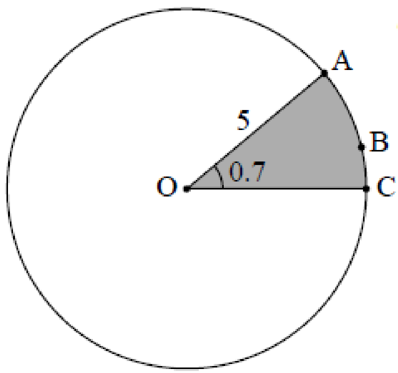


$\mathrm{AD} = 7\text{ cm},\ \mathrm{BC} = 8\text{ cm},\ \mathrm{CD} = 12\text{ cm},\ \mathrm{D\hat{A}B} = 1.75\text{ radians},\ \mathrm{A\hat{B}D} = 0.82\text{ radians}.$

a. Find BD. [3]

b. Find $\mathrm{D\hat{B}C}$. [3]

The following diagram shows a circle with centre O and radius 5 cm.



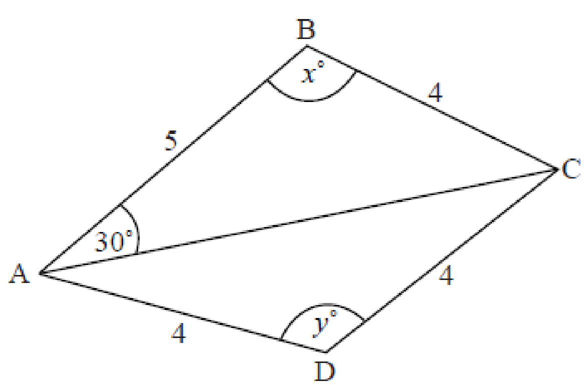
The points A, rmB and rmC lie on the circumference of the circle, and $\mathrm{A\hat{O}C} = 0.7$ radians.

a(i).Find the length of the arc ABC. [2]

a(ii)Find the perimeter of the shaded sector. [2]

b. Find the area of the shaded sector. [2]

The diagram below shows a quadrilateral ABCD with obtuse angles $\mathrm{A\widehat{B}C}$ and $\mathrm{A\widehat{D}C}$.



AB = 5 cm, BC = 4 cm, CD = 4 cm, AD = 4 cm , $\mathrm{B\widehat{A}C} = 30^\circ$, $\mathrm{A\widehat{B}C} = x^\circ$, $\mathrm{A\widehat{D}C} = y^\circ$.

a. Use the cosine rule to show that $\mathrm{AC} = \sqrt{41 - 40\cos x}$. [1]

b. Use the sine rule in triangle ABC to find another expression for AC. [2]

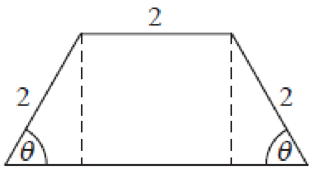
c. (i) Hence, find x, giving your answer to two decimal places. [6]

(ii) Find AC .

d(i) and (ii)(i) Find y. [5]

(ii) Hence, or otherwise, find the area of triangle ACD.

The diagram below shows a plan for a window in the shape of a trapezium.



Three sides of the window are 2 m long. The angle between the sloping sides of the window and the base is θ , where $0 < \theta < \frac{\pi}{2}$.

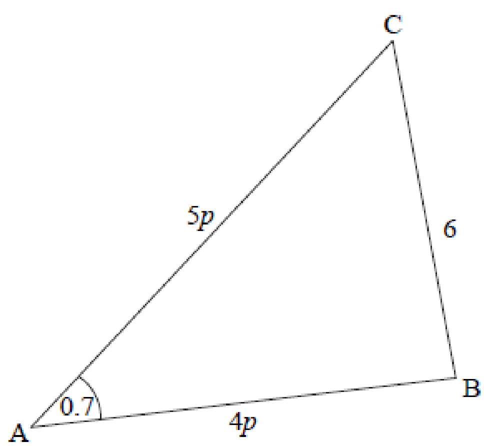
a. Show that the area of the window is given by $y = 4\sin\theta + 2\sin 2\theta$. [5]

b. Zoe wants a window to have an area of 5 m^2. Find the two possible values of θ . [4]

c. John wants two windows which have the same area A but different values of θ . [7]

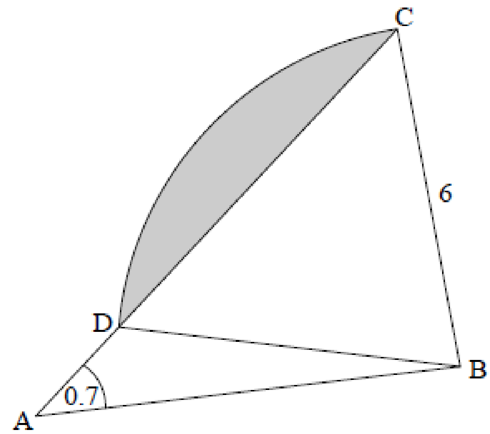
Find all possible values for A .

The following diagram shows a triangle ABC.



$\text{BC} = 6$, $\text{C}\widehat{\text{A}}\text{B} = 0.7$ radians , $\text{AB} = 4p$, $\text{AC} = 5p$, where $p > 0$.

Consider the circle with centre B that passes through the point C. The circle cuts the line CA at D, and $\text{A}\widehat{\text{D}}\text{B}$ is obtuse. Part of the circle is shown in the following diagram.



a(i) and (ii). (i) Show that $p^2(41 - 40\cos 0.7) = 36$. [4]

(ii) Find p .

b. Write down the length of BD. [1]

c. Find $\mathrm{A\widehat{D}B}$. [4]

d(i) and (ii). (i) Show that $\mathrm{C\widehat{B}D} = 1.29$ radians, correct to 2 decimal places. [6]

(ii) Hence, find the area of the shaded region.

Consider the following circle with centre O and radius 6.8 cm.

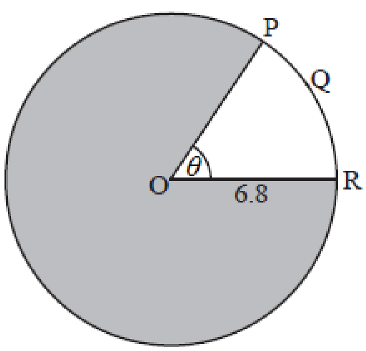


diagram not to scale

The length of the arc PQR is 8.5 cm.

a. Find the value of θ . [2]

b. Find the area of the shaded region. [4]

The diagram shows a parallelogram ABCD.

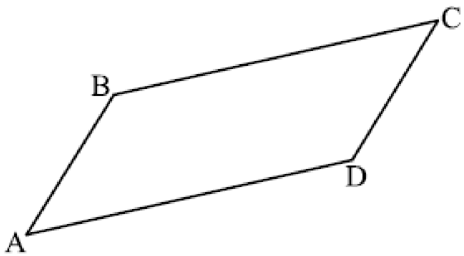


diagram not to scale

The coordinates of A, B and D are A(1, 2, 3) , B(6, 4,4) and D(2, 5, 5) .

a(i), (ii) and (iii). [5]

(i) Show that $\overrightarrow{AB} = \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix}$.

(ii) Find $\overrightarrow{AD}$.

(iii) **Hence** show that $\overrightarrow{AC} = \begin{pmatrix} 6 \\ 5 \\ 3 \end{pmatrix}$.

b. Find the coordinates of point C. [3]

c(i) and (ii). [7]

(i) Find $\overrightarrow{AB} \bullet \overrightarrow{AD}$.

(ii) **Hence** find angle A.

d. Hence, or otherwise, find the area of the parallelogram. [3]

The diagram below shows triangle PQR. The length of [PQ] is 7 cm , the length of [PR] is 10 cm , and $P\widehat{Q}R$ is 75° .

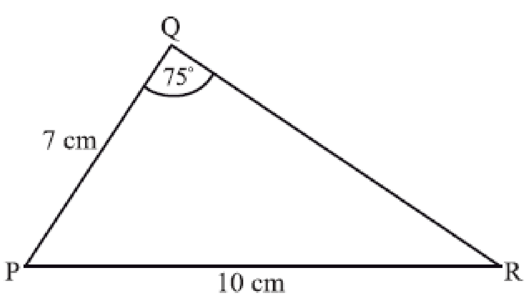
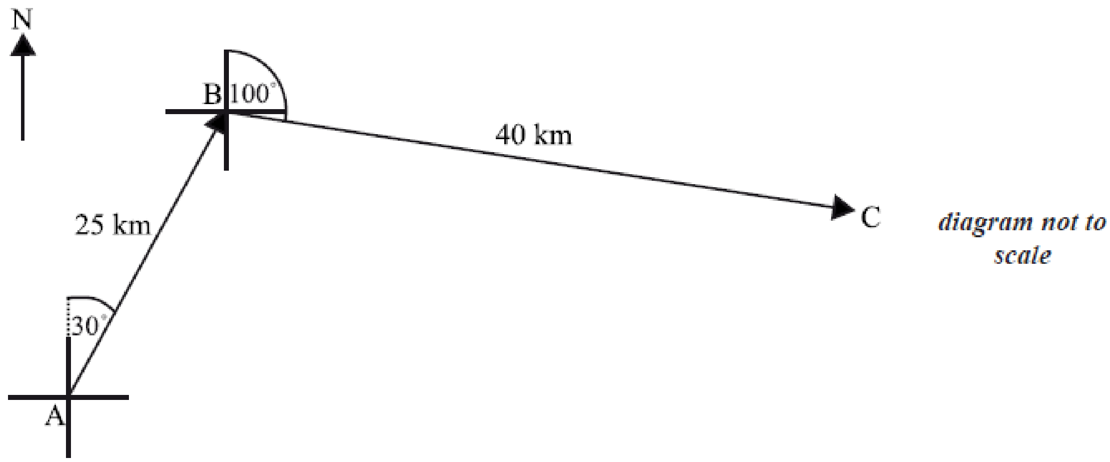


diagram not to scale

a. Find $P\widehat{R}Q$. [3]

b. Find the area of triangle PQR. [3]

A ship leaves port A on a bearing of 030° . It sails a distance of 25 km to point B. At B, the ship changes direction to a bearing of 100° . It sails a distance of 40 km to reach point C. This information is shown in the diagram below.

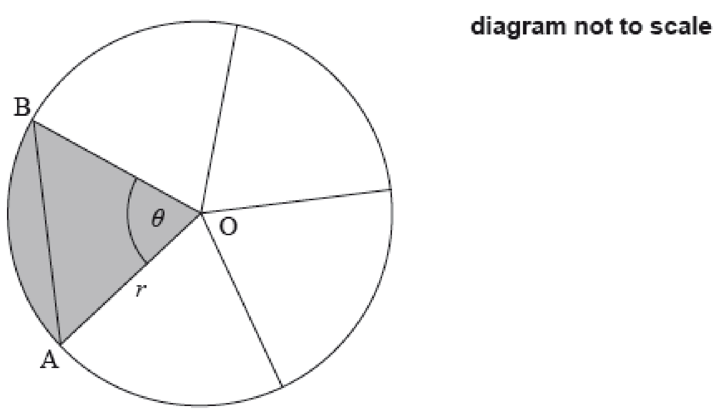


A second ship leaves port A and sails directly to C.

a. Find the distance the second ship will travel. [4]

b. Find the bearing of the course taken by the second ship. [3]

The following diagram shows a circle, centre O and radius r mm. The circle is divided into five equal sectors.



One sector is OAB, and $\mathrm{A\hat{O}B} = \theta$.

The area of sector AOB is $20\pi\ \mathrm{mm}^2$.

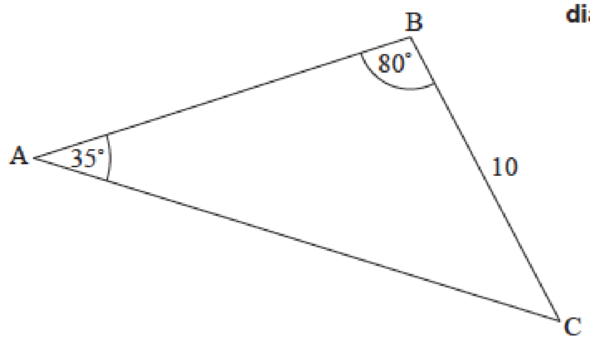
a. Write down the **exact** value of θ in radians. [1]

b. Find the value of r. [3]

c. Find AB. [3]

The following diagram shows triangle ABC.

diagram not to scale

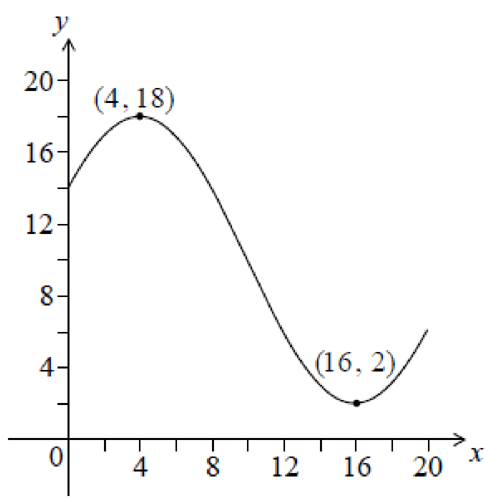


$\mathrm{BC} = 10\text{ cm}$, $\mathrm{A\hat{B}C} = 80°$ and $\mathrm{B\hat{A}C} = 35°$.

a. Find AC. [3]

b. Find the area of triangle ABC. [3]

Let $f(x) = p\cos(q(x+r)) + 10$, for $0 \leqslant x \leqslant 20$. The following diagram shows the graph of f.



The graph has a maximum at $(4, 18)$ and a minimum at $(16, 2)$.

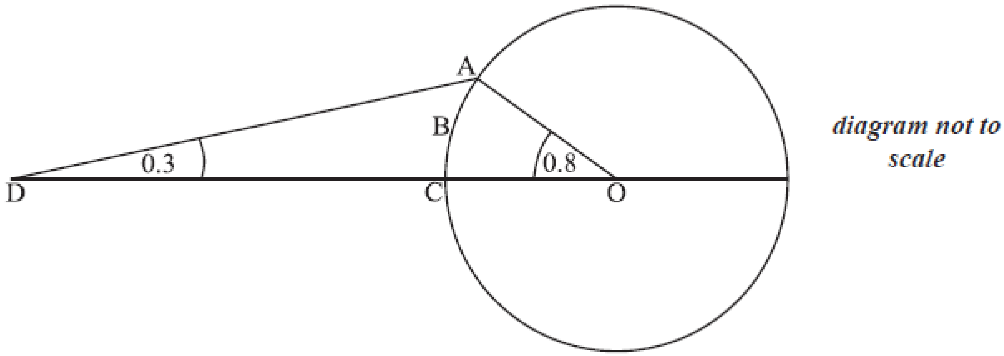
a. Write down the value of r. [2]

b(i). Find p. [2]

b(ii). Find q. [2]

c. Solve $f(x) = 7$. [2]

The following diagram shows a circle with centre O and radius 4 cm.



The points A, B and C lie on the circle. The point D is outside the circle, on (OC).

Angle ADC = 0.3 radians and angle AOC = 0.8 radians.

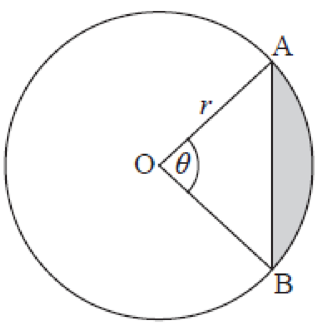
a. Find AD. [3]

b. Find OD. [4]

c. Find the area of sector OABC. [2]

d. Find the area of region ABCD. [4]

A circle centre O and radius r is shown below. The chord [AB] divides the area of the circle into two parts. Angle AOB is θ .



a. Find an expression for the area of the shaded region. [3]

b. The chord [AB] divides the area of the circle in the ratio 1:7. Find the value of θ . [5]

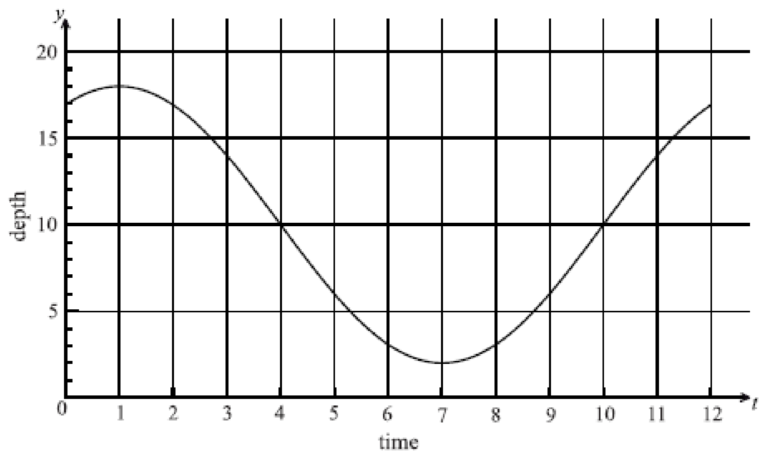
The population of deer in an enclosed game reserve is modelled by the function $P(t) = 210\sin(0.5t - 2.6) + 990$, where t is in months, and $t = 1$ corresponds to 1 January 2014.

a. Find the number of deer in the reserve on 1 May 2014. [3]

b(i). Find the rate of change of the deer population on 1 May 2014. [2]

b(ii). Interpret the answer to part (i) with reference to the deer population size on 1 May 2014. [1]

The following graph shows the depth of water, y metres , at a point P, during one day. The time t is given in hours, from midnight to noon.



a(i), (ii) and (iii). Use the graph to write down an estimate of the value of t when [3]

(i) the depth of water is minimum;

(ii) the depth of water is maximum;

(iii) the depth of the water is increasing most rapidly.

b(i), (ii) and (iii). The depth of water can be modelled by the function $y = \cos A(B(t-1)) + C$. [6]

(i) Show that $A = 8$.

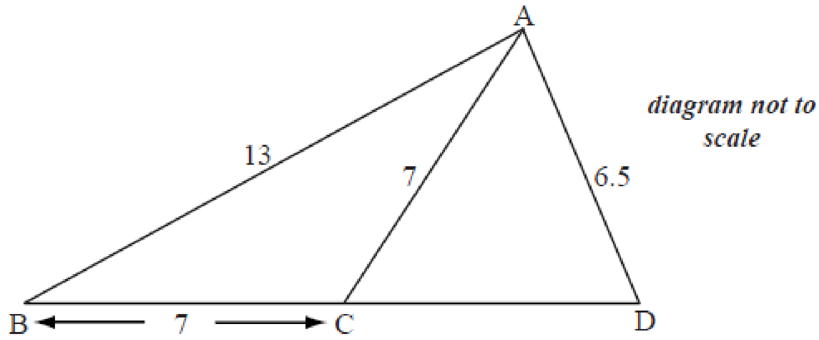
(ii) Write down the value of C.

(iii) Find the value of B.

c. A sailor knows that he cannot sail past P when the depth of the water is less than 12 m . Calculate the values of t between which he cannot sail past P. [2]

The diagram below shows a triangle ABD with AB =13 cm and AD = 6.5 cm.

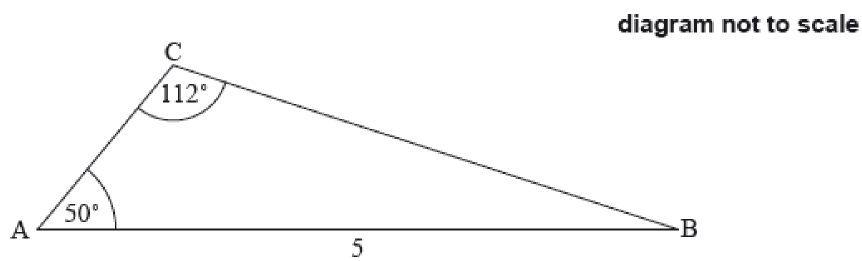
Let C be a point on the line BD such that BC = AC = 7 cm.



a. Find the size of angle ACB. [3]

b. Find the size of angle CAD. [5]

The following diagram shows a triangle ABC.

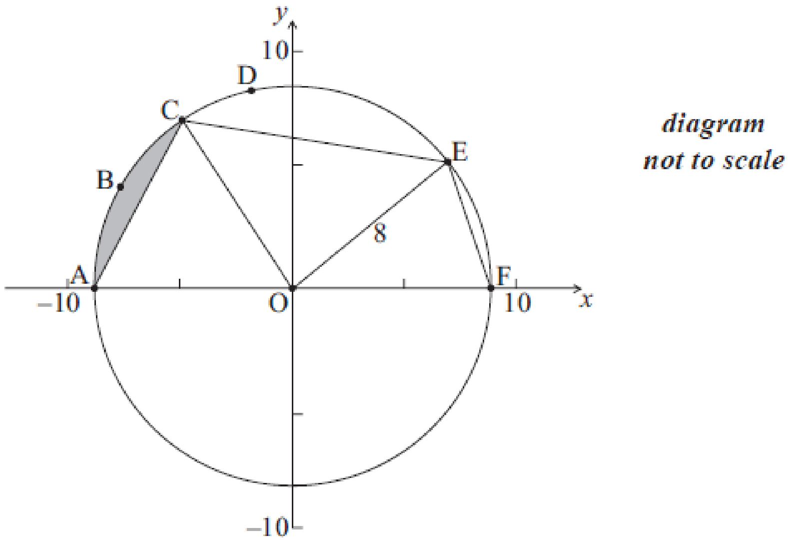


$\mathrm{AB} = 5\mathrm{cm}, \mathrm{C\hat{A}B} = 50°$ and $\mathrm{A\hat{C}B} = 112°$

a. Find BC. [3]

b. Find the area of triangle ABC. [3]

The diagram below shows a circle with centre O and radius 8 cm.



The points A, B, C, D, E and F are on the circle, and [AF] is a diameter. The length of arc ABC is 6 cm.

a. Find the size of angle AOC . [2]

b. Hence find the area of the shaded region. [6]

c. The area of sector OCDE is 45 cm^2. [2]

Find the size of angle COE .

d. Find EF . [5]

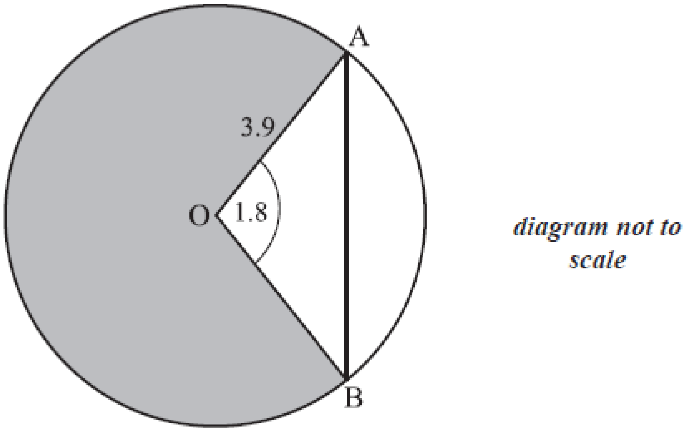
Let $f(x) = \frac{3x}{2} + 1$, $g(x) = 4\cos\left(\frac{x}{3}\right) - 1$. Let $h(x) = (g \circ f)(x)$.

a. Find an expression for $h(x)$. [3]

b. Write down the period of h . [1]

c. Write down the range of h . [2]

The circle shown has centre O and radius 3.9 cm.

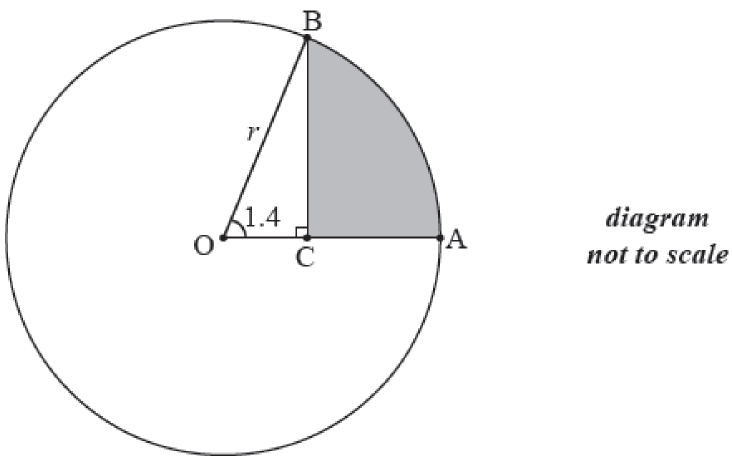


Points A and B lie on the circle and angle AOB is 1.8 radians.

a. Find AB. [3]

b. Find the area of the shaded region. [4]

The following diagram shows a circle with centre O and radius r cm.



Points A and B are on the circumference of the circle and $\mathrm{A\hat{O}B} = 1.4$ radians .

The point C is on [OA] such that $\mathrm{B\hat{C}O} = \frac{\pi}{2}$ radians .

a. Show that $\mathrm{OC} = r\cos 1.4$. [1]

b. The area of the shaded region is 25 cm^2 . Find the value of r . [7]

The following diagram shows a circle with centre O and radius 3 cm.

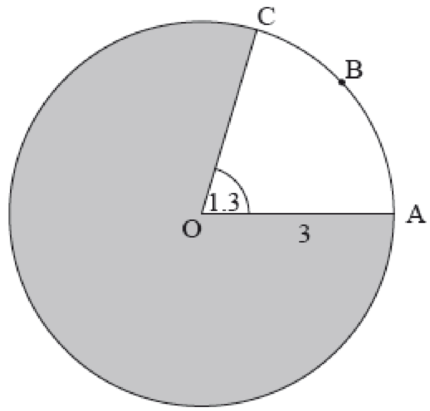


diagram not to scale

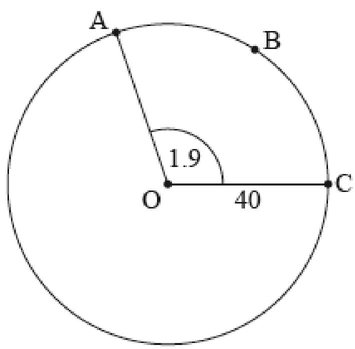
Points A, B, and C lie on the circle, and $\hat{AOC} = 1.3$ radians.

a. Find the length of arc ABC. [2]

b. Find the area of the shaded region. [4]

The following diagram shows a circle with centre O and radius 40 cm.

diagram not to scale



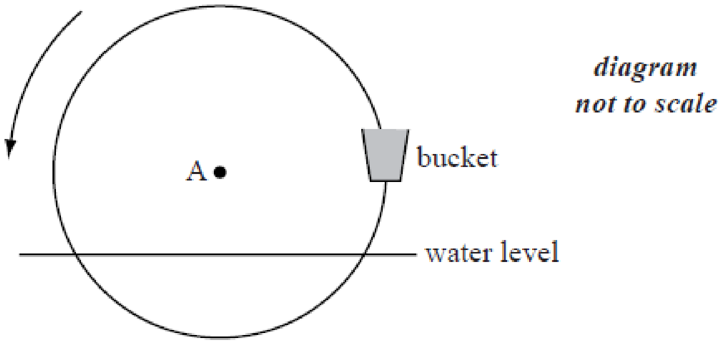
The points A, B and C are on the circumference of the circle and $\hat{AOC} = 1.9$ radians.

a. Find the length of arc ABC. [2]

b. Find the perimeter of sector OABC. [2]

c. Find the area of sector OABC. [2]

The following diagram shows a waterwheel with a bucket. The wheel rotates at a constant rate in an anticlockwise (counter-clockwise) direction.



The diameter of the wheel is 8 metres. The centre of the wheel, A, is 2 metres above the water level. After t seconds, the height of the bucket above the water level is given by $h = a\sin bt + 2$.

a. Show that $a = 4$. [2]

b. The wheel turns at a rate of one rotation every 30 seconds. [2]

Show that $b = \frac{\pi}{15}$.

c. In the first rotation, there are two values of t when the bucket is **descending** at a rate of 0.5 ms^{-1} . [6]

Find these values of t .

d. In the first rotation, there are two values of t when the bucket is **descending** at a rate of 0.5 ms^{-1} . [4]

Determine whether the bucket is underwater at the second value of t .

In triangle ABC, $\text{AB} = 6\text{ cm}$ and $\text{AC} = 8\text{ cm}$. The area of the triangle is 16 cm^2.

a. Find the two possible values for $\hat{A}$. [4]

b. Given that $\hat{A}$ is obtuse, find BC. [3]

Let $f(x) = \cos\left(\frac{\pi}{4}x\right) + \sin\left(\frac{\pi}{4}x\right),\ \text{for } -4 \leqslant x \leqslant 4$.

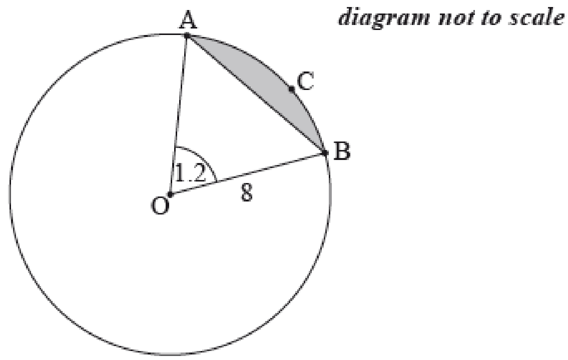
a. Sketch the graph of f. [3]

b. Find the values of x where the function is decreasing. [5]

c(i).The function f can also be written in the form $f(x) = a\sin\left(\frac{\pi}{4}(x+c)\right)$, where $a \in \mathbb{R}$, and $0 \leqslant c \leqslant 2$. Find the value of a; [3]

c(ii).The function f can also be written in the form $f(x) = a\sin\left(\frac{\pi}{4}(x+c)\right)$, where $a \in \mathbb{R}$, and $0 \leqslant c \leqslant 2$. Find the value of c. [4]

The following diagram shows a circle with centre O and radius 8 cm.



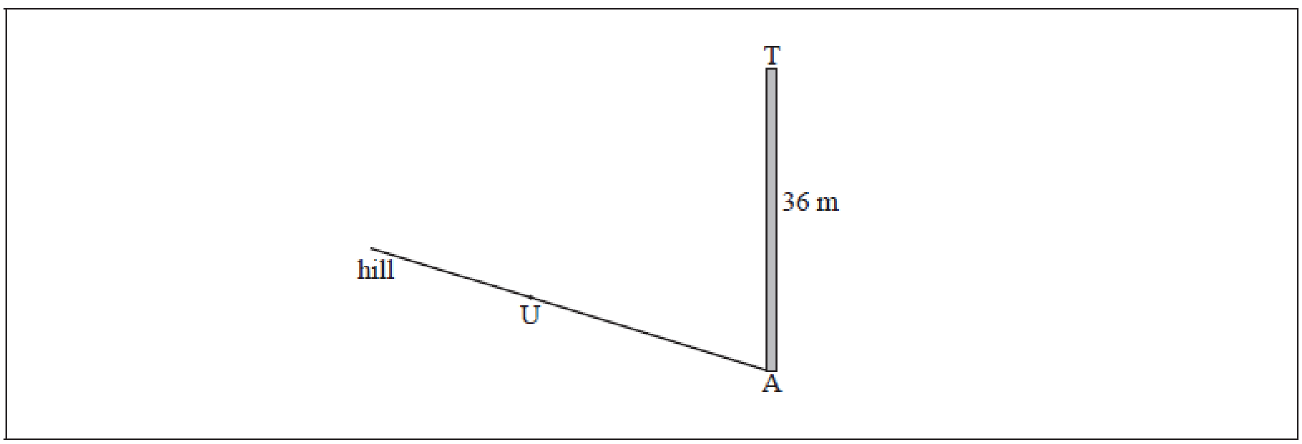
The points A, B and C are on the circumference of the circle, and $\mathrm{A\hat{O}B}$ radians.

a. Find the length of arc ACB. [2]

b. Find AB. [3]

c. Hence, find the perimeter of the shaded segment ABC. [2]

There is a vertical tower TA of height 36 m at the base A of a hill. A straight path goes up the hill from A to a point U. This information is represented by the following diagram.



The path makes a 4° angle with the horizontal.

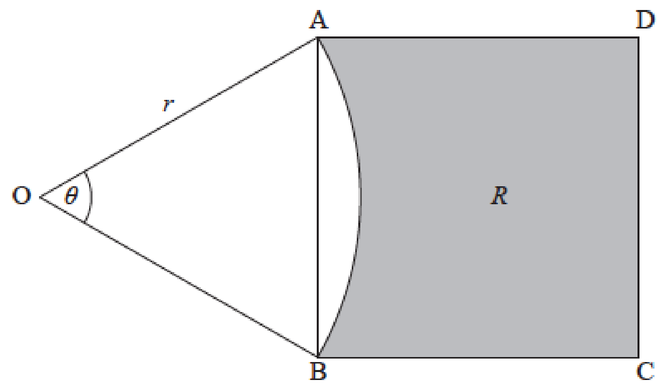
The point U on the path is 25 m away from the base of the tower.

The top of the tower is fixed to U by a wire of length x m.

a. Complete the diagram, showing clearly all the information above. [3]

b. Find x . [4]

The following diagram shows a square $ABCD$, and a sector OAB of a circle centre O, radius r. Part of the square is shaded and labelled R.



$\text{A}\hat{\text{O}}\text{B} = \theta$, where $0.5 \leq \theta < \pi$.

a. Show that the area of the square $ABCD$ is $2r^2(1 - \cos\theta)$. [4]

b. When $\theta = \alpha$, the area of the square $ABCD$ is equal to the area of the sector OAB. [4]

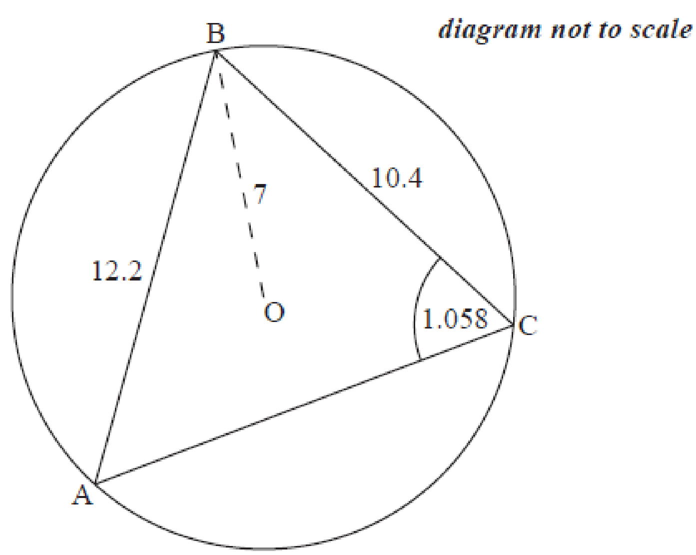
(i) Write down the area of the sector when $\theta = \alpha$.

(ii) Hence find α.

c. When $\theta = \beta$, the area of R is more than twice the area of the sector. [8]

Find all possible values of β.

Consider a circle with centre O and radius 7 cm. Triangle ABC is drawn such that its vertices are on the circumference of the circle.



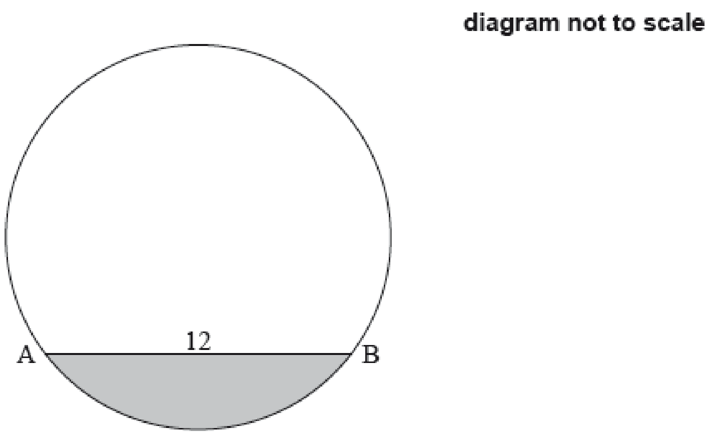
$\text{AB} = 12.2$ cm, $\text{BC} = 10.4$ cm and $\text{A}\hat{\text{C}}\text{B} = 1.058$ radians.

a. Find $\text{B}\hat{\text{A}}\text{C}$. [3]

b. Find AC. [5]

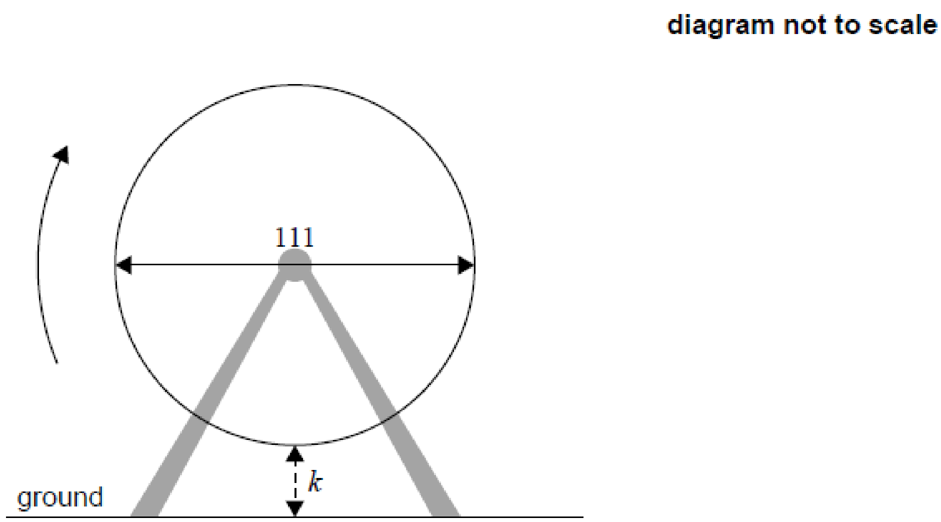
c. Hence or otherwise, find the length of arc ABC. [6]

The following diagram shows the chord [AB] in a circle of radius 8 cm, where $\mathrm{AB} = 12\ \mathrm{cm}$.



Find the area of the shaded segment.

At an amusement park, a Ferris wheel with diameter 111 metres rotates at a constant speed. The bottom of the wheel is *k* metres above the ground. A seat starts at the bottom of the wheel.



The wheel completes one revolution in 16 minutes.

After *t* minutes, the height of the seat above ground is given by $h\left(t\right) = 61.5 + a\cos\left(\frac{\pi}{8}t\right)$, for $0 \le t \le 32$.

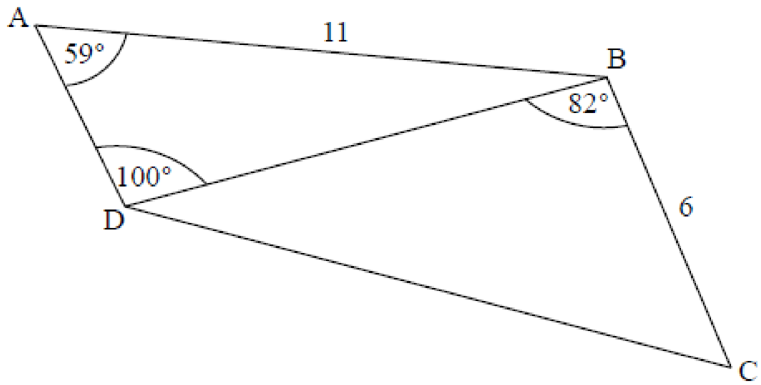
a. After 8 minutes, the seat is 117m above the ground. Find *k*. [2]

b. Find the value of *a*. [3]

c. Find when the seat is 30m above the ground for the third time. [3]

The following diagram shows quadrilateral ABCD.

diagram not to scale

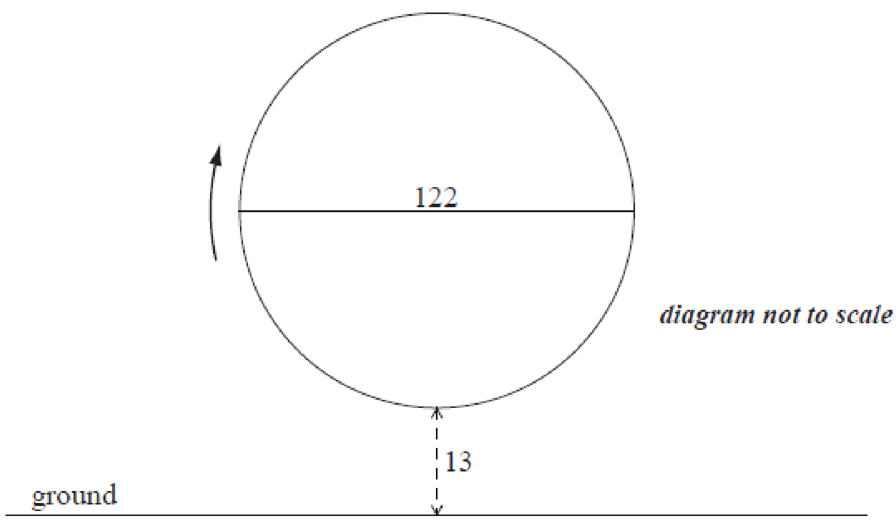


$\mathrm{AB} = 11\,\mathrm{cm},\ \mathrm{BC} = 6\,\mathrm{cm},\ \mathrm{B}\hat{\mathrm{A}}\mathrm{D} = 100^\circ$, and $\mathrm{C}\hat{\mathrm{B}}\mathrm{D} = 82^\circ$

a. Find DB. [3]

b. Find DC. [3]

A Ferris wheel with diameter 122 metres rotates clockwise at a constant speed. The wheel completes 2.4 rotations every hour. The bottom of the wheel is 13 metres above the ground.



A seat starts at the bottom of the wheel.

After t minutes, the height h metres above the ground of the seat is given by

$$h = 74 + a\cos bt.$$

a. Find the maximum height above the ground of the seat. [2]

b. (i) Show that the period of h is 25 minutes. [2]

(ii) Write down the **exact** value of b .

bcd(b) (i) Show that the period of h is 25 minutes. [9]

(ii) Write down the **exact** value of b .

(c) Find the value of a .

(d) Sketch the graph of h , for $0 \leq t \leq 50$.

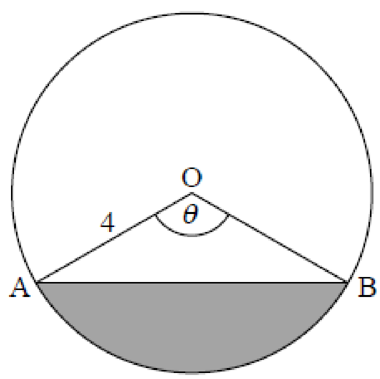
c. Find the value of a . [3]

d. Sketch the graph of h , for $0 \leq t \leq 50$. [4]

e. In one rotation of the wheel, find the probability that a randomly selected seat is at least 105 metres above the ground. [5]

The diagram shows a circle, centre O, with radius 4 cm. Points A and B lie on the circumference of the circle and AÔB = θ , where $0 \leq \theta \leq \pi$.

diagram not to scale

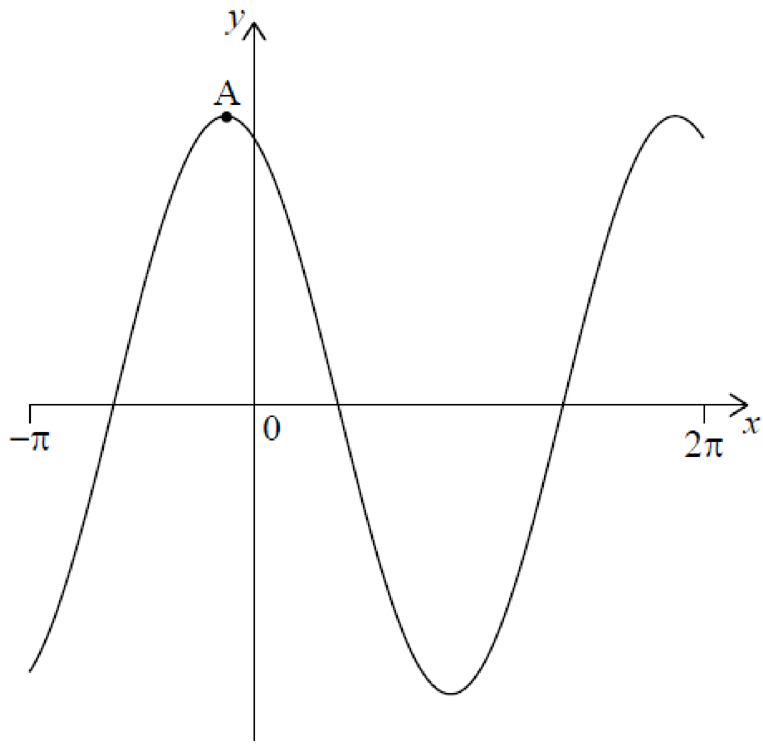


a. Find the area of the shaded region, in terms of θ. [3]

b. The area of the shaded region is 12 cm^2. Find the value of θ. [3]

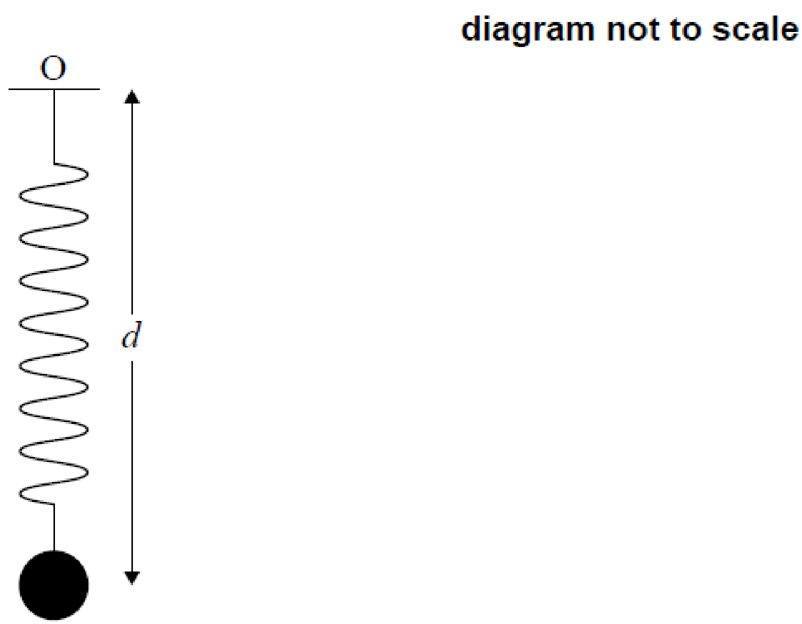
Let $f(x) = 12 \cos x - 5 \sin x, \ -\pi \leqslant x \leqslant 2\pi$, be a periodic function with $f(x) = f(x + 2\pi)$

The following diagram shows the graph of f.



There is a maximum point at A. The minimum value of f is −13 .

A ball on a spring is attached to a fixed point O. The ball is then pulled down and released, so that it moves back and forth vertically.



The distance, *d* centimetres, of the centre of the ball from O at time *t* seconds, is given by

$$d(t) = f(t) + 17,\ 0 \leqslant t \leqslant 5.$$

a. Find the coordinates of A. [2]

b.i. For the graph of f, write down the amplitude. [1]

b.ii. For the graph of f, write down the period. [1]

c. Hence, write $f(x)$ in the form $p\ \cos\ (x + r)$. [3]

d. Find the maximum speed of the ball. [3]

e. Find the first time when the ball's speed is changing at a rate of 2cms^{-2}. [5]